Understanding strategy development in mathematics: using eye movement measurement in educational research

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Abstract

Devices that measure eye movement have made it possible for researchers to obtain and analyze learners’ eye movements when they engage in problem solving. This study investigated if eye movement characteristics differed according to whether strategies were developed during a division puzzle task, and before and after such strategy development. This task entailed inserting numbers in blank boxes while performing a division. Participants needed to think about the structure of the calculation and develop appropriate strategies. Twelve university students were recruited as participants. Our results showed little eye movement when strategies had been developed, and for answers to be completed efficiently. However, when strategies had not been developed, the participants exhibited a tendency toward much eye movement. Further, after developing a strategy, eyes tended to stop moving. We found that eye movement data could reflect the characteristics of the learning processes. Thus, eye movement measurement data could be helpful for understanding the learning processes in younger students as well (such as elementary school students), who often have difficulty explaining their understanding and strategy development conditions. In addition, eye movement measurement could be used to examine the differences in the process of using different strategies for identical problems.

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1. Introduction

For effective instruction in mathematics education, clarifying the processes of understanding in learners is an important research challenge. Discovering the understanding process requires visualization of the learners’ thinking processes in chronological order (Posner, Strike, Hewson, & Gertzog, 1982; Schnitz, Vosniadou, & Carretero, 1999). Three conventional visualization techniques are used in educational research. The first is the “observational technique,” which includes observing learner behavior and performing error analysis. The second is the “interview technique,” which includes face-to-face interviews and protocol acquisition (Compte, Preissle, & Tesch, 1993; Gubrium & Holstein, 2002; Minichiello, Aroni, & Hays, 2008). The third is the “written question technique,” which includes written tests and questionnaires (Gubrium & Holstein, 2002). Some of these techniques can be used in combination with other techniques. In addition, they have been used widely and have exhibited a high degree of reliability.

In recent years, in addition to these conventional techniques of exploring the understanding process of learners, physiological data is being used. Physiological data can be used to acquire chronological information in real-time, yielding highly objective data. One example of physiological data involves brain activity (Ansari & Coch, 2006; Gubrium & Holstein, 2002).
Campbell, 2011; Goswami, 2006; Goswami & Szucs, 2011; Spitzer, 2012; Varma, McCandliss, & Schwartz, 2008; Zamar, Ischebeck, & Delazer, 2009). Studies on elementary school students have exhibited changes in brain activity corresponding to their process of understanding (Kuroda, Chance, & Nioka, 2009; Okamoto & Kuroda, 2011; Okamoto & Maesako, 2009; Rosenberg-Lee, Barth, & Menon, 2011). Similarly, eye movement data is effective in studying the understanding process of learners. Of all the five senses (sight, sound, smell, taste, and touch), vision provides the largest amount of data. Eye movement recording systems have made it comparatively easier to obtain learner vision data, enabling researchers to analyze eye movement characteristics in situations closer to regular learning environments (Kowler, 2011). Further, vision data can be used effectively to analyze learning conditions, because it can measure what the learners are focusing on and can evaluate the characteristics of their eye movements chronologically (Eitel, Scheiter, Schüler, Nyström, & Holmqvist, 2013; Hyönä, 2010; Jarodzka, van Gog, Dorr, Scheiter, & Gerjets, 2013; Jian, Wu, & Su, 2014; Lai et al., 2013).

The purpose of this study was to measure the eye movements during the process of solving division puzzles, and to analyze their characteristics. Particular attention was paid to strategy development, and the differences in eye movements before and after the development of strategies were examined.

2. Method

2.1. Participants

Twelve right-handed university students participated in this experiment (age: Mean = 21.08 years, SD = 0.95; 4 female). None of the participants had any prior history of neurological, psychological, and/or psychiatric disorders, such as ADHD or autism spectrum disorders. They had normal or corrected-to-normal vision. During the experiment, the participants with correct vision were allowed to use only soft contact lenses. No malfunction of the eye-tracking system and failure of calibration was reported during the data collection. Before starting, the details of the experiment were explained to all the participants. In addition, instructions for safety during the experiment and the treatment of the experimental results were explained. Participants signed informed consent forms for participation, and authorized the researchers to use the results from the experiment. Each participant answered a questionnaire after his or her eye movement was measured.

2.2. Eye movement recording

We used the EMR-9 (NAC Image Technology, Japan) eye-tracking system to record the participants’ eye movement. This system consisted of glasses with built-in cameras that were capable of identifying what participants saw and of measuring their eye movements (Figure 1, left). To record what participants saw, the view angle of the camera was 62 degrees. This tracking system produced a video showing the fixation point on the images seen by participants (Figure 1, right). The system applied the corneal reflection method that detected the position of the Purkinje image and the visual axis (Jóźwik, Siedlecki, & Zająć, 2013). A data sampling rate of 60Hz was used, and the automatic calibration procedure included nine calibration points (three points by three points).
2.3. The experimental task

A division puzzle was used as the experimental task. The participants had to insert numbers in blank boxes while performing the mathematical division task. Unlike conventional calculations in which prescribed steps can be followed to determine solutions, this task required the participants to determine answers while thinking about the calculation structure of the division, making it easy to observe and identify their eye fixation and movement characteristics. The task also involved the development and use of strategies for solving the puzzle. Thus, the task also facilitated the examination of the differences in eye movements before and after strategy development.

The task was used in two trials, each having one problem. The problems for Trials (i) and (ii) were 9613/26 and 9241/37 respectively. By balancing the number of digits used and including carry-over operations in the calculation, it was ensured that the two problems were of equal difficulty. Each problem had eight blank boxes placed according to the positions of the digits used while performing a division. The location of the blank boxes was the same for all of the problems. The answer sheets were of A3 size, and were used in landscape orientation. Participants entered their answers by writing with a pen.

3. Results

3.1. Accuracy and strategy

The video recordings and questionnaire confirmed that all 12 participants (Participant A through L) were able to complete all problems within the time limit in both Trials (i) and (ii). With no calculation errors, all participants exhibited 100% accuracy for the two trials.

In terms of strategy development, participants were divided into the following two categories:

- Group that already knew the strategy: 4 participants (Participants A through D)
- Group that did not know the strategy: 8 participants (Participants E through L)

3.2. Eye movement

The eye movement data was used to identify “which blank box in the division puzzle each participant was looking at” or “which section related to which blank box was looked at.” This data was in one-second intervals and
was used in chronological order to analyze eye fixation order and times. As shown in Figure 3, each of the eight blank boxes in the division puzzle were numbered (1) through (8) for classification purposes.

Figures 4 through 7 are graphs of the results obtained using this analysis method. Figure 4 and 5 present the results for two participants who already knew the solution strategy (Participants A and B respectively), and Figures 6 and 7 present the results for two participants who did not know the solution strategy (Participants E and K respectively). The vertical axis indicates the blank boxes that were numbered (1) through (8), which were visually fixated on by the participants, and the horizontal axis indicates the time (in seconds). The solid grey line represents Trial (i), the dotted black line represents Trial (ii), and the circles and triangles on each line indicate the times when answers were written down.

Figure 3. Blank box numbers of the division puzzle

3.2.1. The group that already knew the strategy

Participants in this group showed little eye fixation movement from the start of Trial (i), moving to and stopping on blank boxes one at a time, and spending a certain amount of time on each as illustrated in Figures 4 and 5. Further, eye fixation movement tended to occur after filling in the blank box and there was no eye fixation duplication. In other words, once a blank box was filled in, the participants did not return to those boxes. The eye fixation sequences and answer sequences were identical for both Trial (i) and (ii), producing similar line graphs. The eye fixation and answers progressed in the same order, from blank box (8), to (7), to (6); this was continued in an upward direction.

Figure 4. Eye movement measurement data of Participant A  
Figure 5. Eye movement measurement data of Participant B
3.2.2. The group that did not know the strategy

From the start of Trial (i), there was a great deal of eye fixation movement in this group, and it was rare for eyes to remain fixed on a single blank box (as illustrated in Figures 6 and 7). Often, the order of eye fixation for Trial (i) was (1), (3), and (4), with frequent movement between these blank boxes. However, once members of this group began entering answers, there was little eye fixation movement and eyes tended to remain fixed on one blank box at a time. This was similar to the group that already knew the strategy. The answer and eye fixation movement sequences were similar for Trials (i) and (ii), but were not necessarily identical. With regard to the answer sequence, unlike the members of the group that already knew the strategy, it was observed that the members of this group filled in the divisor or the blank box (3), which was followed by boxes (1), (4), and (5), and then continued in a downward direction.

![Figure 6: Eye movement measurement data of Participant E](image1)
![Figure 7: Eye movement measurement data of Participant F](image2)

4. Discussion

Depending on the spatial position of the blank boxes, there are several ways of solving problems such as this. However, one highly effective solution strategy is to solve from the bottom up, in an order opposite to the one used in regular calculation. This strategy is effective in helping learners gain an understanding of the calculation structure, which is difficult to grasp when regular mechanical calculation steps are used.

The group that already knew the strategy used it, and immediately entered the answers from the bottom and upwards, without trial-and-error. They did not return to areas they had already answered, and answered with a high level of efficiency. The answer sequences for both Trial (i) and (ii) were identical, so participants appeared to have established the strategy since the beginning of the task.

However, the group that did not know the strategy frequently switched the point of eye fixation between blank box (1), (3), and (4), looking at the same areas as they would during regular calculation, and using trial-and-error. It appears that they were attempting to rapidly discover the divisor in blank box (3), and then fill in the blank boxes using the standard calculation approach. For this group, the answer sequences in Trial (i) and (ii) were similar, but not identical, indicating that the participants were still establishing strategies to solve the problem. None of the participants in this group could discover the strategy of solving the problem in reverse order, so it is believed that this strategy was difficult to discover on its own.

5. Conclusions

An eye movement recording system was used in this arithmetic and mathematical education research to reveal the following two findings, and to identify potential for future research. It was found that 1) when strategies had
already been developed, there was a tendency for little eye movement, and answers were filled in efficiently; and 2) when strategies had not been developed, a higher tendency for eye movement was observed. However, after developing a strategy, eyes tended to stop moving as much, and the same trends were seen as in the first finding.

Participants in this study were university students; however, such eye movement measurement data could be helpful data for understanding aspects of learning in elementary school students as well, who often have difficulty explaining their understanding and strategy development conditions. Eye movement measurement could also be used to explore the differences in the process of using different strategies for identical problems.

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